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For: METHODS AND ARTICLES FOR DETECTING, VERIFYING, AND REPAIRING COLLINEARITY IN A MODEL OR SUBSETS OF A MODEL

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REQUEST FOR CORRECTED PUBLICATION PURSUANT TO 37 C.F.R. 1.221(b)

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Sir:

Pursuant to 37 C.F.R. 1.221(b), we hereby request republication of the above-referenced patent application, which was published as U.S. 2004/0249481-A1 on December 9, 2004, due to the following material mistakes:

Please note that this request is being filed within the two month period allowed.

In paragraph [0043] on page 3, "$ct_i\sigma_1$* mt_i, i=1, . . . m" should be replaced with --$ct_i = \sigma_i * mt_i$, $i = 1,...,m$--. The correct version appears on page 7, line 18 of the specification as filed.

In paragraph [0057] on page 3, "ct_i,i=r+, . . . ,m" should be replaced with --$ct_i, i = r+1,...,m$--. The correct version appears on page 9, line 8 of the specification as filed.

In the last sentence of paragraph [0084] on page 5, "v_{ij}(sm)" should be replaced with --$v_{li}(sm)$--. The correct version appears on page 12, line 12 of the specification as filed.

In paragraph [0087] on page 5, "σ_{ij}^{-}(sm)=σ_{ij}^{+}(sm)=0" should be replaced with --$\sigma_{ii}^{-}(sm) = \sigma_{ii}^{+}(sm) = 0$--. The correct version appears on page 12, line 17 of the specification as filed.

In paragraph [0088] on page 5, "σ_{ij}^{-}(sm)=σ_1(0)*(1-eps) and σ_{ij}^{+}(sm)=σ_i(0)*(1+eps)" should be replaced with --$\sigma_{ii}^{-}(sm) = \sigma_i(0) * (1 - eps)$ and $\sigma_{ii}^{+}(sm) = \sigma_i(0) * (1 + eps)$--. The correct version appears on page 12, line 19 of the specification as filed.

At page 5, paragraph [0097], the line reading "i,j=1, . . . , $m(ms)$ Constraint 11.1" should be deleted. This line does not appear in the specification as filed.

At page 8, paragraph [0145], --with singular-- should be inserted before "values $\sigma = (3.2117 \quad 1.89876 \quad 0.60946 \quad 3.5965e-5)$". The correct version appears on page 20, lines 1-2 of the specification as filed.

Enclosed are copies of the relevant pages of the publication with changes noted in red.

Since the mistakes were made by the U.S. Patent and Trademark Office and not by Applicants or Applicants' Attorney/Agent, it is understood that there are no additional fees for the requested republication.

Respectfully submitted,

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By ______________________

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$$\sigma_1 = \max_{MV \neq 0} \frac{\|CV\|_2}{\|MV\|_2} \text{ and } \sigma_m = \min_{MV \neq 0} \frac{\|CV\|_2}{\|MV\|_2}.$$

[0037] The maximum is achieved when MV moves along the direction defined by V_1 (herein referred to as "the strong direction"), and the minimum is achieved when MV moves along the direction defined by V_m (herein referred to as "the weak direction").

[0038] From Equation 2, each element in G can be expressed by Equation 4:

$$g_{ij} = \sum_{k=1}^{m} \sigma_k * u_{ik} * v_{jk} \quad \text{Equation 4}$$

[0039] For a square system (i.e., where n=m), the relationship defined by Equation 2 can be reversed as shown in Equation 5:

$$\mathrm{Diag}(\sigma_i) = U' * G * V \quad \text{Equation 5}$$

[0040] Given the threshold s>0, a model matrix has a rank of r(s) if:

$$\sigma_r/\sigma_1 >= s \text{ and } \sigma_{r+1}/\sigma_1 < s.$$

[0041] $r(s) \geqq 0$ and $r(s) \leqq m$. If r(s)=m, then the given system has a full rank and the matrix is not "collinear." If r(s)<m and $\sigma_m=0$, the system is "collinear" or "perfectly collinear." If r(s)<m and $\sigma_m=0$, then the system is "nearly collinear."

[0042] A transformed input vector MT and a transformed output vector CT as follows:

$$MT = V' * MV \quad \text{Equation 6}$$

$$CT = U' * CV \quad \text{Equation 6.1}$$

[0043] Then the transformed input and output variables have the following relationship:

$$ct_i = \sigma_i * mt_i, i=1, \ldots, m$$

$$\cancel{ct_i \sigma_i * mt_i\ i=1, \ldots, m} \quad \text{Equation 7}$$

[0044] G(sm) is a square sub-matrix derived from G, where sm=1, . . . , p and p is the number of all possible square sub-matrices. The dimension of G(sm) ranges from 2×2, 3×3, . . . , to m×m. When dealing with collinearity, the focus can be placed on square sub-matrices because if a n×m matrix is collinear (where n>m),then all its m×m sub-matrices must be collinear too.

[0045] Collinearity Detection

[0046] For a given model matrix G, and a given threshold s, a search over all sub-matrices G(sm), where sm=1, . . . , p, is conducted and the sub-matrices are sorted into three groups: 1) a not collinear group, $G_n(sm)$; 2) a nearly collinear group, $G_{nc}(sm)$; and, 3) a collinear group, $G_c(sm)$.

[0047] If the $G_{nc}(sm)$ group is empty, the method can be stopped. Alternatively, the threshold value can be modified and the method of the invention begun again. If the $G_{nc}(sm)$ group is not empty, the collinearity of the sub-matrixes in that group is verified.

[0048] Collinearity Verification

[0049] When a sub-matrix falls into the nearly collinear group, the available degree of freedom is then either recognized or ignored even though the extra capability may be limited. In order to determine the best course of action for a given application, the collinearity can be verified. This verification includes determining whether the control action is acceptably aggressive for the needs of the given application, and controlling as described below:

[0050] Control Action Magnitude

[0051] When a system is nearly collinear, the associated control action for certain CV targets will be aggressive, since the control action is, to some extent, proportional to the response of the model inverse. The most significant magnitude (in the sense of 2-norm) of MV change happens when the CV move along the weak direction, that is,

$$CV = \alpha * U_m, \alpha \in R \quad \text{Equation 8}$$

$$MV = (\alpha/\sigma_m) * V_m \quad \text{Equation 9}$$

[0052] α can be used as a scaling number so that the CV change will be in a desired range, and then the required MV change can be estimated from Equation 9. Based on the knowledge of the process, a practitioner can make a judgment if the control action required to achieve the targets is reasonable. If it is determined that the control action is desirable, then the near collinearity is acceptable and no model adjustment is necessary. If it is determined that the action is not desirable, a directional test and identification is performed.

[0053] Directional Test and Identification

[0054] As shown in Equation 7, for a nearly collinear process, the transformed outputs ct_i,i=r+1, . . . , m will have a relatively small response to the transformed inputs mt_i,i=r+1, . . . , m. Moreover, their corresponding singular value σ_i is similar to the gain between the transformed input and the transformed output. This relationship provides an opportunity to verify if the real process is the case, by employing the following procedure:

[0055] 1. Perturb the system in such a way that the input signals follow the direction specified by U_i,i=r+1, . . . , m.

[0056] 2. Construct transformed input and outputs vectors mt_i and ct_i,i=r+1, . . . , m using the collected test data.

[0057] 3. Identify model between mt_i and ~~ct_i,i=r+1, . . . , m~~. $ct_i, i=r+1, \ldots, m$.

[0058] 4. If the identified gain in the transformed space is close to σ_i and the uncertainty bound of the identified gain surrounds 0, then this portion of the process is deemed to be truly collinear; otherwise, it is deemed to be not collinear.

[0059] 5. The newly collected test data is used to improve the model quality by rerunning the identification because the newly collected data contains rich process responses in the weak direction.

[0060] 6. Repeat the collinearity detection with the improved model.

[0080] To address these concerns, the following optimization formula is created:

$$\text{Minimize}\sum_{i=1}^{n}\sum_{j=1}^{m}(g_{ij}-g_{ij(0)})^2 \qquad \text{Equation 10}$$

[0081] subject to:

$$g_{ij}^- =< g_{ij} <= g_{ij}^+, i=1, \ldots, n, j=1, \ldots, m \qquad \text{Constraint 10.1}$$

[0082]

$$\sigma_{ij}^-(sm) = < \sum_{k=1}^{m(sm)}\sum_{l=1}^{m(sm)} g_{lk}(sm) * u_{li}(sm) * v_{kj}(sm) <= \sigma_{ij}^+(sm) \qquad \text{Constraint 10.2}$$

$$i,j=1, \ldots, m(sm), sm=1, \ldots, p$$

[0083] G(0) represents the nominal model, G^+ and G^- represent the allowed upper and lower bound of the model. σ_{ij}^- and σ_{ij}^+ are allowed upper and lower bound for the singular values. m(sm) is the dimension of the sub-matrix sm. p is the total number of nearly collinear sub-matrices $G_{nc}(sm)$.

[0084] Equation 10 is the objective function. The objective fumction minimizes deviation from the nominal model as long as the required perfect collinearlity can be achieved. Constraint 10.1 represents the allowed variation for each model element. Constraint 10.2 is contributed from each nearly collinear sub-matrix. All $u_{1i}(sm)$ and $V_{lj}(sm)$ ~~$v_{1j}(sm)$~~ are the singular vectors calculated from the original sub-matrix $G_{nc}(sm)$, and hence the same directionality is always maintained.

[0085] In some embodiments, σ_{ij}^- and σ_{ij}^+ are set to

[0086] 1) $\sigma_{ij}^-(sm)=\sigma_{ij}^+(sm)=0$ if $i \neq j$, which corresponds to the off-diagonal portion of the singular value matrix;

$\sigma_{ii}^-(sm)=\sigma_{ii}^+(sm)=0$

[0087] 2) ~~$\sigma_{ij}^-(sm)=\sigma_{ij}^+(sm)=0$~~ if i>r(sm), which corresponds to those small singular values to be zeroed out;

$\sigma_{ii}^-(sm)=\sigma_i(0)*(1-eps)$ and $\sigma_i^+(sm)=\sigma_i(0)*(1+eps)$

[0088] 3) ~~$\sigma_{ij}^-(sm)=\sigma_i(0)*(1-eps)$ and $\sigma_{ij}^+(sm)=\sigma_i(0)*(1+eps)$~~ if i<=r(sm), where 0<eps<1 is a constant. Choosing a large value for eps allows large variations in the singular values. Since the objective function always tries to find the minimal variation for the model matrix, it is expected that variation of the singular value will also be very mild. Hence, a small eps (for instance, eps=0.1) can be safely used.

[0089] Finally, Equation 10 is a standard QP formula and can be solved globally and efficiently.

[0090] Uncollinearization

[0091] If the process is not collinear, or is nearly collinear but needs the controller to fully explore its capability, then adjustments to the model can be made to improve the condition number so that an improved robustness can be achieved.

[0092] Uncollinearization should satisfy the following requirements:

[0093] 1. The repaired model should have the same directionality as the original model because changing the direction can cause unwanted control problems that can potentially result in a less desirable performance than with the original collinearity.

[0094] 2. The directionality change should be made within allowed ranges. Additional restrictions can also be imposed, such as additional linear equality or inequality constraints.

[0095] 3. When treating a model matrix larger than 2×2, a collinear sub-matrix can share common elements with another collinear sub-matrix. Such a case can result in a "zigzag game" or never ending loop, with repairs to one sub-matrix giving rise to a need for repairs to the other sub-matrix. Hence, the methodology should be able to deal with multiple sub-matrices in a synchronized way.

[0096] To achieve these goals, the following optimization formula was created:

$$\text{Maximize}\sum_{sm=1}^{p}\sum_{i=1}^{m(sm)-r(sm)}(\sigma_{r(sm)+i}/\sigma_{r(sm)+i}(0))^2 \qquad \text{Equation 11}$$

[0097] subject to:

$$g_{ij}^+(sm) = < \sum_{k=1}^{m(sm)}\sigma_k(sm) * u_{ik}(sm) * v_{jk}(sm) <= g_{ij}^-(sm), \qquad \text{Constraint 11.1}$$

$$i, j = 1, \ldots, m(ms)$$

~~i,j=1, . . . , m(ms) Constraint 11.1~~

$$\sigma_i^-(sm) =< \sigma_i(sm) <= \sigma_i^+(sm), i=1, \ldots, m(sm) \qquad \text{Constraint 11.2}$$

$$\sum_{k=1}^{m(q)}\sigma_k(q) * u_{ik}(q) * v_{jk}(q) = \sum_{k=1}^{m(t)}\sigma_k(t) * u_{ik}(t) * v_{jk}(t), \qquad \text{Constraint 11.3}$$

[0098] for those i,j which point to the same element in G, sm, q, t=1, . . . ,p, $q \neq t$.

[0099] g_{ij}^+ and g_{ij}^- denote the upper and lower bounds of the allowed model adjustment, σ_i^- and σ_i^+ are upper and lower bounds on the singular values, $\sigma_{r+i}(0)$ represents the original singular value, and m(sm) is the dimension of the sub-matrix sm. p is the total number of nearly collinear sub-matrices $G_{nc}(sm)$. Additional explanation of Equation 11 is provided below.

[0100] The objective function is to maximize the portion of the smaller singular values of all nearly collinear sub-matrices. The weighting factor, $1/\sigma_{r(sm)+i}(0)$, means the smaller the original singular value, the more improvement the optimizer will attempt to obtain. Constraint 11.1 represents the allowed variation for each model element. All $u_{ik}(sm)$ and $v_{jk}(sm)$ (as well as $u_{ik}(q)$, $v_{jk}(q)$, $u_{ik}(t)$, and

[0141] Let's consider a gain matrix shown below:

$$G=\begin{bmatrix} -0.6112 & -0.8705 & -0.8161 & -2.2515 & 0 & 0 & 0 & 0 & 0 & 0 & 4.4013 & 0 \\ -2.8114 & -5.3374 & -10.1706 & -30.0073 & 1.8984 & 1.8283 & 0.1132 & 0.0153 & 0.1204 & 0.0495 & 0 & 0 \\ -2.0609 & 6.0295 & 9.4035 & 24.5094 & -1.6758 & -1.7359 & -0.154 & -0.0253 & -0.1818 & -0.0677 & 0 & 0 \\ -0.6244 & -1.4399 & -2.5429 & -7.4598 & 0.4203 & 0.4147 & 0 & 0.0091 & 0.0456 & 0.0187 & 0 & -0.2577 \\ 0.1446 & 0.3026 & 0.5424 & 1.5888 & -0.0855 & -0.0843 & 0 & -0.0016 & -0.0105 & -0.004 & 0 & 0.1572 \\ -1.7149 & -2.731 & 6.9034 & 17.5966 & -1.2014 & -1.2893 & 0 & -0.0407 & -0.1277 & -0.0382 & 0 & 0 \\ -0.0015 & -0.0034 & 0.004 & 0.0227 & 0 & -0.0016 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[0142] Given a threshold of s=0.0001, the following three sub-matrices are identified to be collinear:

$$G_1=\begin{bmatrix} 6.0295 & -1.7359 \\ -1.4399 & 0.4147 \end{bmatrix},$$

[0143] with singular values σ=(0.831627 7.57818e−5)

$$G_2=\begin{bmatrix} -1.6758 & -0.1818 \\ -0.4203 & 0.0456 \end{bmatrix},$$

[0144] with singular values σ=(3.44311 6.69126e−5)

$$G_3=\begin{bmatrix} -2.8114 & 1.8984 & 1.8283 & 0.0153 \\ -2.0609 & -1.6758 & -1.7359 & -0.0253 \\ -0.6244 & 0.4203 & 0.4147 & 0.0091 \\ -1.7149 & -1.2014 & -1.2893 & -0.0407 \end{bmatrix},$$

with singular

[0145] values σ=(3.2117 1.89876 0.60946 3.5965e−5)

[0146] Assuming that each gain element is allowed to vary no more than 10% from its nominal value.

[0147] The Collinearization results:

$$G_1=\begin{bmatrix} 6.0302 & -1.73614 \\ -1.44011 & 0.414617 \end{bmatrix},$$

[0148] with singular values σ=(0.831509 0)

$$G_2=\begin{bmatrix} -1.67583 & -0.181804 \\ -0.420308 & 0.0455975 \end{bmatrix},$$

[0149] with singular values σ=(3.44302 0)

$$G_3=\begin{bmatrix} -2.81033 & 1.89821 & 1.82815 & 0.0153128 \\ -2.0605 & -1.67583 & -1.73614 & -0.0254196 \\ -0.62416 & 0.420308 & 0.414617 & 0.00902449 \\ -1.71455 & -1.20166 & -1.28927 & -0.0405464 \end{bmatrix},$$

[0150] with singular values σ=(3.21072 1.89861 0.602891 0)

[0151] The uncollinearization result:

$$G_1=\begin{bmatrix} 5.56824 & -1.59648 \\ -1.32195 & 0.408456 \end{bmatrix},$$

[0152] with singular values σ=(0.905079 0.0169216)

$$G_2=\begin{bmatrix} -1.5367 & -0.1818 \\ -0.414725 & 0.0456 \end{bmatrix},$$

[0153] with singular values σ=(3.64152 0.0666536)

$$G_3=\begin{bmatrix} -3.09168 & 1.8772 & 1.79803 & 0.01377 \\ -2.11308 & -1.5367 & -1.59648 & -0.02277 \\ -0.68684 & 0.414725 & 0.408456 & 0.00819 \\ -1.76702 & -1.09372 & -1.18305 & -0.0400757 \end{bmatrix},$$

[0154] with singular values σ=(3.23766 1.9333 0.648371 0.00239357).

What is claimed is:

1. A method of analyzing a model for Model Predictive Control, the method comprising the steps of:

a) obtaining a model gain matrix of a subject model used for Model Predictive Control of a given process;

b) identifying any near-collinear sub-matrices of the obtained model gain matrix;